EDM Note

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Comment on Systematic Error in Deuteron EDM Measurement Due to Tensor Polarization

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Ed Stephenson has pointed out a systematic error in the deuteron edm measurment:

"The sensitivity of nuclear reactions to polarization is described by a set of scattering angle dependent functions called analyzing powers. For the spin-1 deuteron, there are one vector and three tensor analyzing powers. In spherical tensor notation, the vector is $iT_{11}(\theta)$ and the tensors are $T_{20}(\theta)$, $T_{21}(\theta)$, and $T_{22}(\theta)$ where θ is the angle at which the exiting particle is observed. If we describe the orientation of the positive direction along the quantization axis by two polar coordinate angles, β the polar angle with respect to the beam direction, and φ the azimuthal angle with its zero along the y-axis, perpendicular to the scattering plane, the scattering cross section is given by

$$\underline{\sigma(\theta)} = \sigma_0(\theta)\Big(1 + \underline{\sqrt{3}\, p_z \sin\beta \cos\varphi\, iT_{11}(\theta)} + \frac{1}{2\sqrt{2}} p_{zz} (3\cos^2\beta - 1) T_{20}(\theta)$$
$$+ \underline{\sqrt{3}\, p_{zz} \sin\beta \cos\beta \sin\varphi\, T_{21}(\theta)} - \frac{\sqrt{3}}{4} p_{zz} \sin^2\beta \cos 2\varphi\, T_{22}(\theta)\Big).$$

where $\sigma_0(\theta)$ is the cross section measured with an unpolarized beam.

The term containing iT_{11} generates the EDM signal we want to measure. The precession can be upward toward the positive y-axis, thus $\cos\varphi = 1$. The $\sin\beta$ factor represents the EDM precession. This has been given as 0.12 mrad after a 1 sec store for a limit of $d = 4 \times 10^{-25}$ e-cm. If we assume that the deuteron beam has only vector polarization, then there is an upper limit on p_z of 2/3. A reasonable value would be $p_z = 0.6$. The asymmetry we seek to measure becomes

$$(\sqrt{3})(0.6)(1.2 \times 10^{-4})(0.23) = 2.9 \times 10^{-5}$$

A systematic error contribution arises from the term containing T_{21} to the measurement of a left-right asymmetry. This is because, like the $\cos\varphi$ dependence of the iT_{11} term, there is a $\sin\varphi$ factor that can change sign on opposite sides of the beam. In this case p_{zz} must have some non-vanishing value."

Since β is a small angle, we get for the time dependent left-right asymmetry, to first order:

$$\underline{\sin\beta \approx \beta} \quad \underline{\sin^2\beta \approx 0} \quad \underline{\cos\beta \approx 1}$$

$$A_{LR}(t) = T_{11} p_z \beta \cos\varphi + T_{21} p_{zz} \beta \sin\varphi$$

Yuri Orlov has pointed out that, in Cartesian coordinates, the first term corresponds to a measurement of the rotation of the spin into the vertical plane about the radial axis, which is the edm effect we wish to measure. The second term corresponds to a measurement of the rotation of the spin from longitudinal polarization towards transverse polarization about the vertical axis: "Taking into account that β<<1, we get that the (edm) value we need to observe is proportional to $p_z T_{11} dS_y$, as $dS_y = \sin\beta \times \cos\varphi$, while the contamination part of our observation is proportional to $p_{zz} T_{21} dS_x$ as $dS_x = \sin\beta \times \sin\varphi$", where we use dS to mean $d\hat{S}$.

$$\underline{dA_{LR} = d\frac{N_L - N_R}{N_L + N_R} = p_z T_{11} dS_y + p_{zz} T_{21} dS_x}$$

Ed suggests taking the tensor polarization as $p_{zz} \approx 10^{-2}$ and $T_{21} \approx T_{11}$. The p_{zz} term is small, but the edm term is even smaller: zero in the case of no edm. By symmetry, counters placed above and below the median plane for a predominantly vector polarized beam measure:

$$\underline{dA_{UD} = d\frac{N_U - N_D}{N_U + N_D} = -p_{zz} T_{21} dS_y + p_z T_{11} dS_x} \quad \approx p_z T_{11} dS_x$$

This can be written in matrix form:

$$\begin{matrix} dA_{LR} \\ \\ dA_{UD} \end{matrix} = \begin{pmatrix} Z_{11} & Z_{21} \\ \\ -Z_{21} & Z_{11} \end{pmatrix} \begin{matrix} dS_y \\ \\ dS_x \end{matrix}$$

The spin precession axis due to the edm is about the vector $\left(-\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B}\right)$.

$$\vec{\omega} = -\frac{e}{m}\left[a\vec{B} + \left(\frac{1}{\gamma^2 - 1} - a\right)\frac{\vec{\beta} \times \vec{E}}{c} + \frac{\eta}{2}\left(-\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B}\right)\right]$$

where the above equation is accurate for $\beta \bullet B = \beta \bullet E = 0$. The precession axis due to the magnetic anomaly is about the vector:

$$a\vec{B} + \left(\frac{1}{\gamma^2 - 1} - a\right)\frac{\vec{\beta} \times \vec{E}}{c}$$

With incomplete cancellation of the first and second terms in the above equation, there will be a net g-2 precession. We show in Fig. 1 these three terms for when the spin is originally polarized at -90 degrees with respect to the beam direction, and allowed to precess to +90 degrees, and in

Fig. 2 for when the g-2 precession is allowed five turns over the same time period. The Lorentz force changes by a negligible amount in these two cases[1]. The very small asymmetry $p_{zz}T_{21}dS_y$ contribution to dA_{UD} is not shown.

Fig. 1. Three spin asymmetries for small non-zero edm value and predominantly vector polarized beam. The small asymmetries are enlarged for clarity.

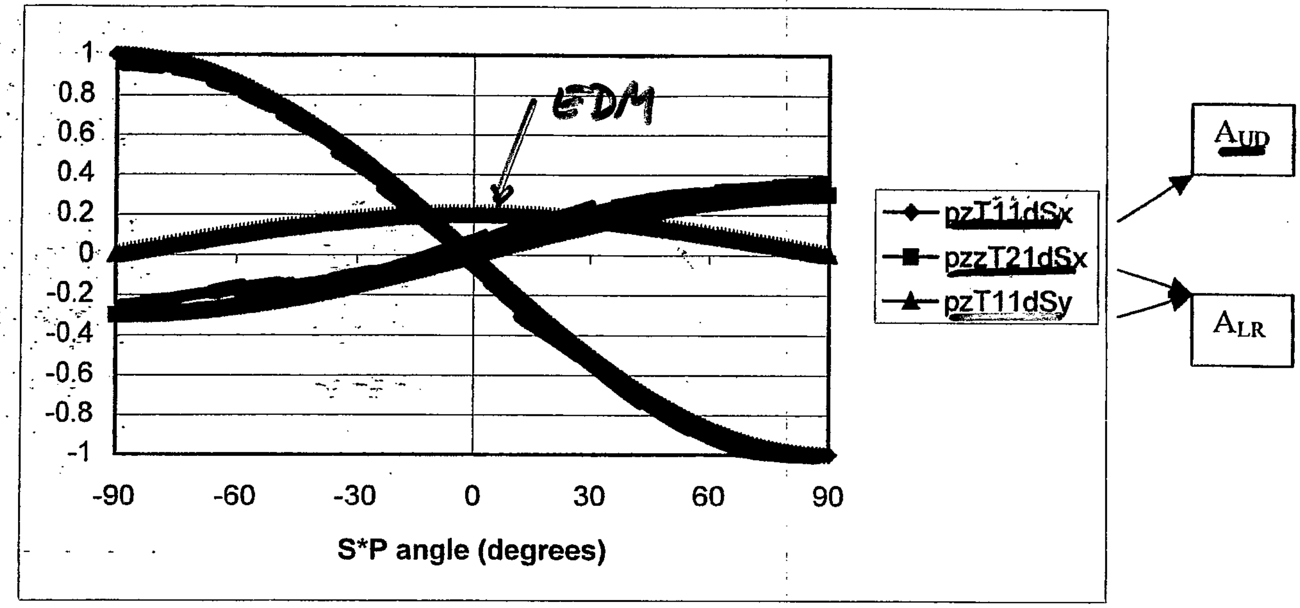
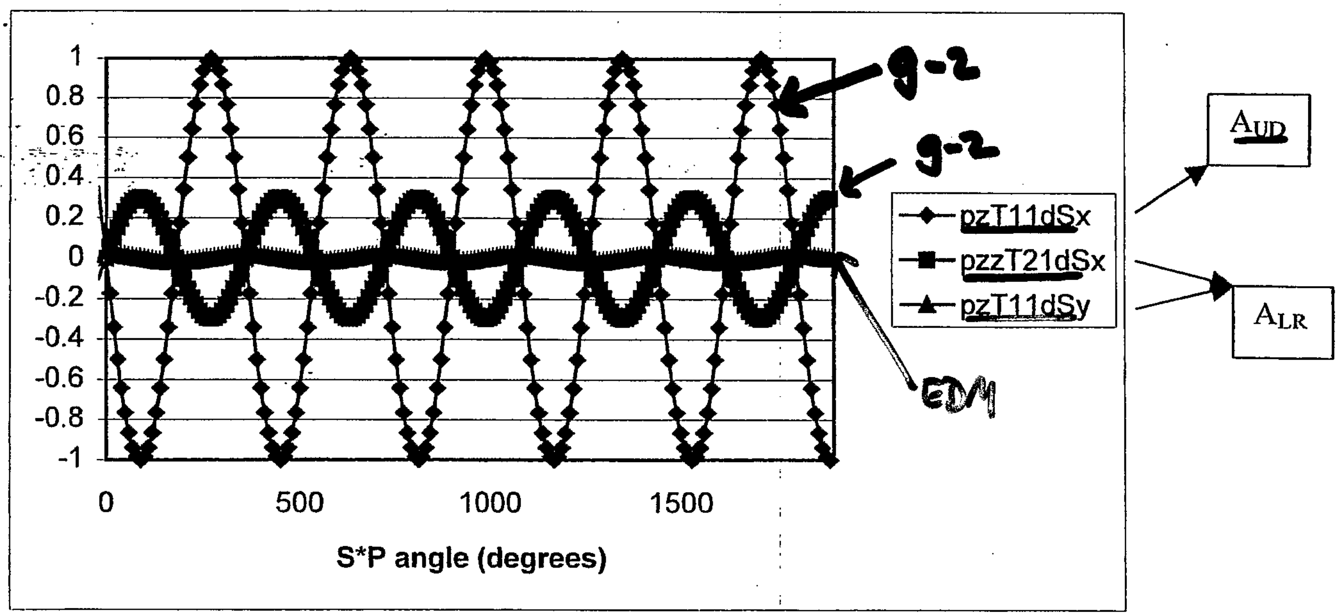


Fig. 2. Three spin asymmetries for small non-zero edm value and predominantly vector polarized beam when the spin precesses five turns in the same time period as Fig.1. Note the edm effect is much smaller.



The edm effect is much smaller in Fig. 2 because the spin repeatedly rotates through the precession axis (E/c + $\beta \times B$). We now give the prescription for subtracting the tensor systematic effect from the edm signal.

First we fit the data in Fig. 2 to determine f:

$$A_{UD}(t) = A_1 \sin(\omega t + \varphi)$$

$$A_{LR}(t) = A_2 \sin(\omega t + \varphi)$$

$$f = \frac{A_2}{A_1}$$

In actuality, the polarization will be decreasing with time and ω will itself be a function of time, but a clever graduate student will still be able to extract f from the data. Then we calculate the edm signal from the data in Fig. 1:

$$S_{EDM}(t) = A_{LR}(t) - f \times A_{UD}(t)$$

There is a factor of √2 loss of statistical power since half the running is spent with a large number g-2 oscillations to determine f. This should probably be on alternate pulses, to properly average over temporal changes in source p_{zz}/p_z. Thus we need to run longer for the same edm statistical power, but we have solved the systematic error associated with the tensor polarization of the deuteron beam.

References

1. W. Morse, EDM Note 31.